Lecture 38

Module IV - Failure Theory of Composites

Failure Theory of Composites

For isotropic material, the load carrying capacity of a structure can be determined from the principal stresses and ultimate tensile, compressive and shear strengths since the elastic constants of an isotropic material are direction independent. However, in the case of orthotropic materials, the strengths and elastic constants are direction dependent. Hence, different strength values can be obtained for an orthotropic material depending upon the direction of the application of load. More over uniaxial stress applied in any direction other than the principal material axes produces multiaxial sstrains along the principal material axes of orthotropic material. Therefore, the strengths of orthotropic materials must be predicted through an appropriate failure criterion.

Many failure theories are not general but are applicable only to some specific types of composites. In this module, the theories used for fiber composites will be discussed. To use these theories, applied stresses/strains are transformed into the corresponding stresses/strains along the principal material directions.

Maximum-Stress Failure Theory:

It states that failure will occur if any one of the stresses (induced by the applied loads) in the principal material axes exceed the corresponding allowable stress. Therefore, to avoid failure all the following inequalities must be satisfied.

$$\sigma_L < \sigma_{LU} \tag{5.9}$$

$$\sigma_T < \sigma_{TU} \tag{5.10}$$

$$\tau_{LT} < \tau_{LTU} \tag{5.11}$$

where, σ_L, σ_T, and τ_{LT} are the stresses produced by the applied loads and

σ_{LU}, σ_{TU}, and τ_{LTU} are the allowable stresses.

Here, it is to be noted that when the applied normal stress is compressive then the appropriate allowable compressive stress must be used. Further, there is no interaction between the failure modes; that is why all the inequalities should be satisfied simultaneously to avoid failure.

While solving problem, the applied stresses (i.e. σ_x , σ_y etc) are to be resolved into the stresses along the principal material directions (i.e. σ_L , σ_T etc.) using stress transformation law. If an orthotropic lamina is subjected to a stress σ_x making an angle θ with the longitudinal direction, the stresses in the principal material direction are (using the stress transformation)

$$\sigma_L = \sigma_x \cos^2\theta \tag{5.12}$$

$$\sigma_T = \sigma_x \sin^2\theta \tag{5.13}$$

$$\tau_{LT} = -\sigma_x \sin\theta\cos\theta \tag{5.14}$$

It can be concluded from the equations (5.1) and (5.2) that the applied stress σ_x should be the least of the following stress values to avoid failure:

$$\sigma_x < \frac{\sigma_{LU}}{\cos^2\theta} \tag{5.15}$$

$$\sigma_x < \frac{\sigma_{TU}}{\sin^2\theta} \tag{5.16}$$

$$\sigma_x < \frac{\tau_{LTU}}{\sin\theta\cos\theta} \tag{5.17}$$

On the other hand, if the applied stress σ_x does not satisfy any of the above conditions then the following failures will occur.

if, $\sigma_x \geq \frac{\sigma_{LU}}{\cos^2\theta}$ then, longitudinal fiber failure (5.18)

if, $\sigma_x \geq \frac{\sigma_{TU}}{\sin^2\theta}$ then, transverse failure (5.19)

if, $\sigma_x \geq \frac{\tau_{LTU}}{\sin\theta\cos\theta}$ then, shear failure (5.20)

Thus, the safe value of σ_x depends on the fiber orientation angle θ.

At small values of θ, the longitudinal tensile failure will occur and in this case the lamina strength is calculated from $\frac{\sigma_{LUt}}{\cos^2\theta}$.

At high values of θ, the transverse tensile failure will occur and the lamina strength is calculated from $\frac{\sigma_{TUt}}{\sin^2\theta}$.

At intermediate values of θ, the in-plane shear failure will occur and the lamina strength is calculated from the expression $\frac{\tau_{LTU}}{\cos\theta\sin\theta}$.

Maximum-Strain Failure Theory:

It states that failure will occur if any one of the strains (due to the applied loads) in the principal material direction exceeds the corresponding allowable strain. Therefore, to avoid failure the following inequalities must be satisfied.

$$\varepsilon_L < \varepsilon_{LU} \tag{5.21}$$

$$\varepsilon_T < \varepsilon_{TU} \tag{5.22}$$

$$\gamma_{LT} < \gamma_{LTU} \tag{5.23}$$

where, ε_L , ε_T , and γ_{LT}

are the strains, due to the applied loads and

ε_{LU} , ε_{TU} , and γ_{LTU} are the allowable strains

As already stated in the maximum stress theory, when the normal strain due to the applied load is compressive (shortening) then the appropriate allowable compressive strain must be used. Again, as in the case of previous theory here also there is no interaction between the failure modes, that is why all the inequalities should be satisfied simultaneously to avoid failure.

If the material is linearly elastic up to the ultimate failure, the allowable strains can be replaced by the corresponding strength values, as given below:

$$\varepsilon_{LU} = \frac{\sigma_{LU}}{E_L} \tag{5.24}$$

$$\varepsilon_{TU} = \frac{\sigma_{TU}}{E_T} \quad (5.25)$$

$$\gamma_{LTU} = \frac{\tau_{LTU}}{G_{LT}} \quad (5.26)$$

If an orthotropic lamina, with fiber orientation angle of θ with the longitudinal direction, is subjected only to stress σ_x, the strains in the principal material directions are (using the strain-stress relations and the equation (5.2))

$$\varepsilon_L = \frac{1}{E_L}(\cos^2\theta - \nu_{LT}\sin^2\theta)\sigma_x \quad (5.27)$$

$$\varepsilon_T = \frac{1}{E_T}(\sin^2\theta - \nu_{TL}\cos^2\theta)\sigma_x \quad (5.28)$$

$$\gamma_{LT} = \frac{1}{G_{LT}}(\sin\theta\cos\theta)\sigma_x \quad (5.29)$$

It can be concluded from the equations (5.5) and (5.6) that the applied stress σ_x should be the least of the followings to avoid failure:

$$\sigma_x < \frac{\sigma_{LU}}{(\cos^2\theta - \nu_{LT}\sin^2\theta)} \quad (5.30)$$

$$\sigma_x < \frac{\sigma_{TU}}{(\sin^2\theta - \nu_{TL}\cos^2\theta)} \quad (5.31)$$

$$\sigma_x < \frac{\tau_{LTU}}{\sin\theta\cos\theta} \quad (5.32)$$

As the material is assumed to be elastically linear up to the ultimate failure, both the maximum-stress theory and the maximum-strain theory will lead to almost an identical result, but due to the Poisson ratio involved in the latter theory, there may be a slight difference. When the material is not linearly elastic up to failure, both the theories will predict differently.

Tsai-Hill Failure Theory:

This theory provides a single criterion to predict the failure of a lamina. It states that under plane stress condition the failure will occur when the following inequality is satisfied.

$$\left(\frac{\sigma_{11}}{\sigma_{LU}}\right)^2 - \left(\frac{\sigma_{11}\sigma_{22}}{\sigma_{LU}^2}\right) + \left(\frac{\sigma_{22}}{\sigma_{TU}}\right)^2 + \left(\frac{\tau_{12}}{\tau_{LTU}}\right)^2 \geq 1 \qquad (5.33)$$

The above relation is valid if the material is transversely isotropic. If the material is orthotropic and the state of stress is plane stress condition the criterion is

The point to remember here is that this theory will not provide the information about the mode of failure as in the case of maximum stress/strain theory because the interactions of the modes of failure are taken into account. This theory considers the interaction of strength values. The limitation of this theory is that it does not consider the compressive strength values of the lamina.

If an off axis load σ_x is applied on the laamina, the criterion becomes

Tsai-Wu Failure Theory:

This theory provides a single criterion to predict the failure of lamina. It states that under plane stress condition the failure will occur when the following inequality is not satisfied.

$$A_1\sigma_L + A_{11}\sigma_L^2 + A_2\sigma_T + A_{22}\sigma_T^2 + 2A_{12}\sigma_L\sigma_T + A_{66}\tau_{LT}^2 < 1 \qquad (5.34)$$

where, $A_1 = \left(\frac{1}{\sigma_{LU}}\right) - \left(\frac{1}{\sigma'_{LU}}\right)$ (5.35)

$A_{11} = \left(\frac{1}{\sigma_{LU}\sigma'_{LU}}\right)$

$A_2 = \left(\frac{1}{\sigma_{TU}}\right) - \left(\frac{1}{\sigma'_{TU}}\right)$

$A_{22} = \left(\frac{1}{\sigma_{TU}\sigma'_{TU}}\right)$

$A_{12} = -\frac{1}{2}\sqrt{A_{11}A_{22}}$

$A_{66} = \frac{1}{\tau_{LTU}^2}$

The general expression of Tsai-Wu criterion is obtained by substituting the values of Egn. (5.35) in Egn. (5.34).

Failure prediction for delamination initiation

The delamination or ply separation is due to inter-laminar stresses, which can reduce the failure strength of laminas.

In order to predict the initiation of determination at the free edges where inter-laminar stresses are vulnerable, the following quadratic failure criterion is suggested.

$$\frac{\overline{\sigma_z^2}}{\sigma_{zUt}^2}+\frac{\overline{\tau_{xz}^2}}{\tau_{xzU}^2}+\frac{\overline{\tau_{yz}^2}}{\tau_{yzU}^2}=1 \quad (5.36)$$

where, the average inter-laminar stresses are defined by

$$\left(\overline{\sigma_z},\overline{\tau_{xz}},\overline{\tau_{yz}}\right)=\frac{1}{x_c}\int_0^{x_c}\left(\sigma_z,\tau_{xz},\tau_{yz}\right)dx \quad (5.37)$$

$$x_c = 2*t, \quad (5.38)$$

critical distance over which the inter-laminar stresses are averaged.

Consequence of Lamina failure:

After a lamina fails, the stresses and strains in the remaining laminae increase and the laminate stiffness is reduced. For the analysis purpose the following methods can be considered.

(i) Total discount method → In this method, once a lamina fails then its stiffness and strength in all directions are assigned to zero.

(ii) Limited discount method → Zero stiffness and strength are assigned to failed lamina for the transverse and shear modes if the lamina failure is in the matrix material. If the lamina fails by fiber rupture, the total discount method is adopted.

(iii) Residual property method → In this method, residual strength and stiffness are assigned to the failed lamina.

Problems:

The material properties are, E_1= 147 GPa, E_2= 15 GPa, G_{12}= 12 GPa and ν_{12} = 0.3 . For the lamina with orientation [45^o], find the lamina stresses due to the load of N_{xx} = 100 kN/m. Verify for failure through the different failure criteria, if the strength values are

σ_{LU} = 1200 MPa

σ_{TU} = 60 MPa

τ_{LTU} = 90 MPa

References:

"Mechanics of Composite Structural Elements", H Altenbach, J Altenbach and W Kissing, Springer publications.

B D Agarwal, L J Broughtman and K Chandrashekhara, "Analysis and Performance of Fiber Composites", John Wiley and Sons, Inc

Lectures 39 and 40

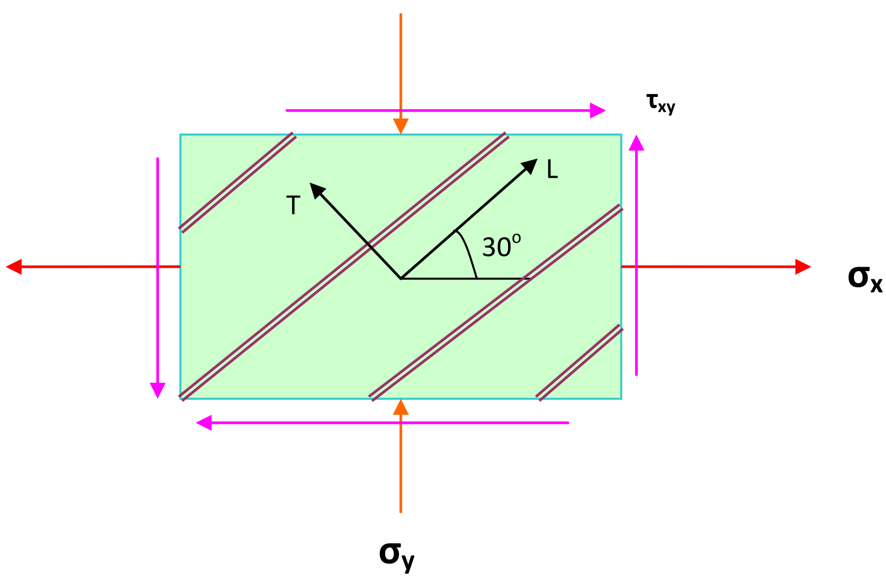
Problems (Module IV):

Problem 5.1: A unidirectional composite lamina is subjected to stresses as shown in Fig. 5.1. It has the allowable tensile stress of 750 MPa in the fiber direction and 50 MPa in the fiber transverse direction and the allowable compressive stress of 400 MPa in the fiber direction and 100 MPa in the fiber transverse direction. The allowable shear stress is 50 MPa. Determine whether, the lamina will fail under the applied stresses using the maximum-stress theory.

Case 1 : $\sigma_x = 50$ MPa,(Tension),$\sigma_y = 25$ MPa (Comp), $\tau_{xy} = 50$ MPa (+ve)

Case 2 : $\sigma_x = 100$ MPa,(Comp), $\sigma_y = 25$ MPa (Comp), $\tau_{xy} = 50$ MPa (+ve)

Case 3 : $\sigma_x = 50$ MPa,(Comp), $\sigma_y = 150$ MPa (Comp), $\tau_{xy} = 50$ MPa (+ve)



Solution:

Case 1:

Given data:-

The applied stresses,

$\sigma_x = 50$ MPa, (Tension)

σ_y = -25 MPa (Compression)

τ_{xy} = 50 MPa (Positive shear)

The allowable stresses,

σ_{LU} = 750 MPa, (Tension)

σ'_{LU} = 400 MPa (Compression)

σ_{TU} = 50 MPa, (Tension)

σ'_{TU} = 100 MPa (Compression)

τ_{xy} = 50 MPa (Shear)

Fiber Orientation angle, $\theta = 30^o$

Since, the fibers are oriented at 30^o to the x axis, stress-transformation matrix is used to get the stresses along the principal material directions.

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{pmatrix} \cos^2 30^o & \sin^2 30^o & 2\cos30\sin30^o \\ \sin^2 30^o & \cos^2 30^o & -2\cos30\sin30^o \\ -\sin30^o\cos30^o & \sin30^o\cos30^o & \cos^2 30-\sin^2 30^o \end{pmatrix} \begin{Bmatrix} 50 \\ -25 \\ 50 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{pmatrix} \cos^2 30^\circ & \sin^2 30^\circ & 2\cos 30^\circ \sin 30^\circ \\ \sin^2 30^\circ & \cos^2 30^\circ & -2\cos 30^\circ \sin 30^\circ \\ -\sin 30^\circ \cos 30^\circ & \sin 30^\circ \cos 30^\circ & \cos^2 30^\circ - \sin^2 30^\circ \end{pmatrix} \begin{Bmatrix} 50 \\ -25 \\ 50 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{Bmatrix} 74.55 \\ -49.55 \\ -7.48 \end{Bmatrix} \text{MPa} < \begin{Bmatrix} 750 \\ -100\text{ (C)} \\ 50 \end{Bmatrix} \text{MPa} \tag{5.39}$$

Since, all the induced stresses are within the permissible limits, as per the maximum stress theory **the lamina will not fail under this applied loading condition**.

Case 2:

Given data:-

The applied stresses,

σ_x = -100 MPa, (Compression)

σ_y = -25 MPa (Compression)

τ_{xy} = 50 MPa (Positive shear)

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{pmatrix} \cos^2 30^\circ & \sin^2 30^\circ & 2\cos 30^\circ \sin 30^\circ \\ \sin^2 30^\circ & \cos^2 30^\circ & -2\cos 30^\circ \sin 30^\circ \\ -\sin 30^\circ \cos 30^\circ & \sin 30^\circ \cos 30^\circ & \cos^2 30^\circ - \sin^2 30^\circ \end{pmatrix} \begin{Bmatrix} -100 \\ -25 \\ 50 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{Bmatrix} -37.95 \\ -87.05 \\ 57.48 \end{Bmatrix} \text{MPa} \not< \begin{Bmatrix} -400(C) \\ -100(C) \\ 50 \end{Bmatrix} \text{MPa} \quad (5.40)$$

Since, the induced shear stress exceeds the permissible limit, as per the maximum stress theory **the lamina will fail in shear under the applied loading condition.**

Case 3:

Given data:-

The applied stresses,

σ_x = -50 MPa, (Compression)

σ_y = -150 MPa (Compression)

τ_{xy} = 50 MPa (Positive shear)

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{pmatrix} \cos^2 30^\circ & \sin^2 30^\circ & 2\cos 30^\circ \sin 30^\circ \\ \sin^2 30^\circ & \cos^2 30^\circ & -2\cos 30^\circ \sin 30^\circ \\ -\sin 30^\circ \cos 30^\circ & \sin 30^\circ \cos 30^\circ & \cos^2 30^\circ - \sin^2 30^\circ \end{pmatrix} \begin{Bmatrix} -50 \\ -150 \\ 50 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{Bmatrix} -31.7 \\ -168.3 \\ -18.3 \end{Bmatrix} \text{MPa} \nless \begin{Bmatrix} -400(C) \\ -100(C) \\ 50 \end{Bmatrix} \text{MPa}$$

Since, the induced transverse compressive stress exceeds the permissible limit, as per the maximum stress theory **the lamina will experience transverse failure under the applied loading condition.**

Problem 5.2: Determine whether the lamina explained in the problem 5.1 will fail under the same loading conditions using the maximum-strain theory if :

E_L = 40 GPa, E_T = 10 GPa, G_{LT} = 4.5 GPa, ν_{LT} = 0.22

Solution:

The allowable strains are first calculated using equations (1.5).

$$\varepsilon_{LU} = \sigma_{LU} / E_L = 750 / 40 \text{ x } 10^3 = 0.0188 \text{ [stretching]} \quad (5.41)$$

$$\varepsilon'_{LU} = \sigma'_{LU} / E_L = 400/40\times10^3 = 0.010 \text{ [shortening]} \quad (5.42)$$

$$\varepsilon_{TU} = \sigma_{TU} / E_T = 50/10\times10^3 = 0.005 \text{ [stretching]} \quad (5.43)$$

$$\varepsilon'_{TU} = \sigma'_{TU} / E_T = 100/10\times10^3 = 0.010 \text{ [shortening]} \quad (5.44)$$

$$\gamma_{LTU} = \tau_{LTU} / G_{LT} = 50/4.5\times10^3 = 0.011 \text{ [shearing]} \quad (5.45)$$

Case 1:

From the problem 5.1,

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{Bmatrix} 74.55 \\ -49.55 \\ -7.48 \end{Bmatrix} MPa$$

From the stress-strain relationship,

$$\nu_{TL} = \frac{\nu_{LT} E_T}{E_L} \quad (5.46)$$

= 0.22 x 10 / 40

= 0.055

$$\varepsilon_L = \frac{\sigma_L}{E_L} - \frac{\nu_{TL}\sigma_T}{E_T} \qquad (5.47)$$

= 74.55 / 40 x 10^3 – 0.055 x (- 49.55) / 10 x 10^3

= 0.0021

$$\varepsilon_T = \frac{\sigma_T}{E_T} - \frac{\nu_{LT}\sigma_L}{E_L} \qquad (5.48)$$

= (- 49.55) / 10 x 10^3 – 0.22 x 74.55 / 40 x 10^3

= - 0.0054

$$\gamma_{LT} = \frac{\tau_{LT}}{G_{LT}} \qquad (5.49)$$

= (-7.48) / 4.5 x 10^3

= -0.0017

On comparison of strains calculated and allowable strains,

$$\varepsilon_L < \varepsilon_{LU}, \quad \varepsilon_T < \varepsilon'_{TU}, \quad \gamma_{LT} < \gamma_{LTU} \qquad (5.50)$$

Therefore, according to the maximum-strain theory, **the lamina will not fail under the case-1 loading condition.**

Case 2:

From the problem 5.1,

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{Bmatrix} -37.95 \\ -87.05 \\ 57.48 \end{Bmatrix} MPa$$

From the stress-strain relationship,

ε_L = (-37.95) / 40 x 10^3 – 0.055 x (-87.05) / 10 x 10^3

= -0.0005

ε_T = (-87.05) / 10 x 10^3 – 0.22 x (-37.95) / 40 x 10^3

= - 0.0085

γ_{LT} = 57.48 / 4.5 x 10^3

= 0.0128

On comparison of strains calculated and the allowable strains,

$\varepsilon_L \nless \varepsilon_{LU}$, $\varepsilon_T < \varepsilon'_{TU}$, and

$\gamma_{LT} < \gamma_{LTU}$ (5.51)

Therefore, according to the maximum-strain theory, **the lamina will fail under the case-2 loading condition.**

<u>Case 3:</u>

From the problem 5.1,

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{Bmatrix} -31.7 \\ -168.3 \\ -18.3 \end{Bmatrix} \text{MPa}$$

From the stress-strain relationship,

ε_L = (-31.7) / 40 x 10^3 – 0.055 x (-168.3) / 10 x 10^3

= 0.0001

ε_T = (-168.3) / 10 x 10^3 – 0.22 x (-31.7) / 40 x 10^3

= - 0.0167

γ_{LT} = (-18.3) / 4.5 x 10^3

= -0.0041

On comparison of strains calculated and the allowable strains,

$\varepsilon_L < \varepsilon_{LU}$,

$\varepsilon_T \nless \varepsilon'_{TU}$, $\gamma_{LT} < \gamma_{LTU}$

Therefore, according to the maximum-strain theory, **the lamina will fail under the case-3 loading condition.**

Problem 5.3: Redo the problem 5.1 using Tsai-Hill theory.

Solution:

Case 1:

From the problem 5.1,

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{Bmatrix} 74.55 \\ -49.55 \\ -7.48 \end{Bmatrix} \text{MPa}$$

$$\begin{Bmatrix} \sigma_{LU} \\ \sigma'_{TU} \\ \tau_{LTU} \end{Bmatrix} = \begin{Bmatrix} 750 \\ -100(C) \\ 50 \end{Bmatrix} \text{MPa}$$

$$= \left(\frac{74.55}{750}\right)^2 - \left(\frac{74.55*(-49.55)}{750^2}\right) + \left(\frac{-49.55}{-100}\right)^2 + \left(\frac{-7.48}{50}\right)^2$$

$= 0.2843 < 1$ OK.

As it is less than unity, **the lamina will not fail under this load**.

Case 2:

From the problem 5.1,

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{Bmatrix} -37.95 \\ -87.05 \\ 57.48 \end{Bmatrix} \text{MPa}$$

$$\begin{Bmatrix} \sigma'_{LU} \\ \sigma'_{TU} \\ \tau_{LTU} \end{Bmatrix} = \begin{Bmatrix} -400(C) \\ -100(C) \\ 50 \end{Bmatrix} \text{MPa}$$

$$= \left(\frac{-37.95}{400}\right)^2 - \left(\frac{(-37.95)*(-87.05)}{400^2}\right) + \left(\frac{-87.05}{-100}\right)^2 + \left(\frac{57.48}{50}\right)^2$$

$= 2.109 \quad > 1.$ NOT OK.

As it is greater, than unity the **lamina will fail under this load.**

Case 3:

From the problem 5.1,

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{Bmatrix} -31.7 \\ -168.3 \\ -18.3 \end{Bmatrix} \text{MPa}$$

$$\begin{Bmatrix} \sigma'_{LU} \\ \sigma'_{TU} \\ \tau_{LTU} \end{Bmatrix} = \begin{Bmatrix} -400(C) \\ -100(C) \\ 50 \end{Bmatrix} \text{MPa}$$

$$= \left(\frac{-31.7}{400}\right)^2 - \left(\frac{(-31.7)*(-168.3)}{400^2}\right) + \left(\frac{-168.3}{-100}\right)^2 + \left(\frac{-18.3}{50}\right)^2$$

$= 3.006 > 1.$ NOT OK.

As, it is greater than unity the **lamina will fail under this load.**

Problem 5.4: Redo the problem 5.1 using Tsai-Wu theory.

Solution:

From the problem 5.1,

$$\begin{Bmatrix} \sigma_{LU} \\ \sigma'_{LU} \\ \sigma_{TU} \end{Bmatrix} = \begin{Bmatrix} 750 \\ 400 \\ 50 \end{Bmatrix} \text{MPa}$$

Case 1:

$$\begin{Bmatrix} \sigma_{L} \\ \sigma_{T} \\ \tau_{LT} \end{Bmatrix} = \begin{Bmatrix} 74.55 \\ -49.55 \\ -7.48 \end{Bmatrix} \text{MPa}$$

$$A_1 = \left(\frac{1}{\sigma_{LU}}\right) - \left(\frac{1}{\sigma'_{LU}}\right) \quad (5.52)$$

= (1 / 750) – (1 / 400)

= -0.0012

$$A_{11} = \left(\frac{1}{\sigma_{LU}\,\sigma'_{LU}}\right) \quad (5.53)$$

= 1 / (750 x 400)

= $3.33e^{-006}$

$$A_2 = \left(\frac{1}{\sigma_{TU}}\right) - \left(\frac{1}{\sigma'_{TU}}\right) \quad (5.54)$$

= (1 / 50) – (1 / 100)

= 0.010

$$A_{22} = \left(\frac{1}{\sigma_{TU}\ \sigma'_{TU}}\right) \quad (5.55)$$

= 1 / (50 * 100)

= $2.00e^{-004}$

$$A_{12} = -\frac{1}{2}\sqrt{A_{11}A_{22}} \quad (5.56)$$

= - 1/2 ( ) ½

= -1.290 e-005

$$A_{66} = \frac{1}{\tau_{LTU}^2} \quad (5.57)$$

$= 1 / (50)^2$

$= 4.00\ e^{-004}$

$A_1 \sigma_L + A_{11} \sigma_L^2 + A_2 \sigma_T + A_{22} \sigma_T^2 + 2 A_{12} \sigma_L \sigma_T + A_{66} \tau_{LT}^2 = 0.0448 < 1$

Therefore, **the lamina will not fail under this loading condition** according to Tsai-Wu theory.

Case 2:

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{Bmatrix} -37.95 \\ -87.05 \\ 57.48 \end{Bmatrix} \text{MPa}$$

The values of A_1, A_{11}, A_2, A_{22}, A_{12}, and A_{66} are irrespective of the loading condition; therefore, the values calculated in the case 1 will be used here.

$A_1 \sigma_L + A_{11} \sigma_L^2 + A_2 \sigma_T + A_{22} \sigma_T^2 + 2 A_{12} \sigma_L \sigma_T + A_{66} \tau_{LT}^2 = 1.93 \not< 1$

Therefore, **the lamina will fail under this loading condition** according to Tsai-Wu theory.

Case 3:

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{Bmatrix} -31.7 \\ -168.3 \\ -18.3 \end{Bmatrix} \text{MPa}$$

$A_1 \sigma_L + A_{11} \sigma_L^2 + A_2 \sigma_T + A_{22} \sigma_T^2 + 2 A_{12} \sigma_L \sigma_T + A_{66} \tau_{LT}^2 = 4.019 \not< 1$

Therefore, **the lamina will fail under this loading condition** according to Tsai-Wu theory.